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RADIO PHENOMENA
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Application of the Model of Coupled Oscillators in the Analysis of the Nonlinear Excitation of Hypersound in a Ferrite Plate under Ferromagnetic Resonance. Part 1. Basic Equations

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Abstract—The excitation of hypersonic oscillations by ac magnetic field in the configuration with the normally magnetized ferrite plate is considered in the framework of the problem of the magnetostriction microwave transducer. The quadratic approximation with respect to magnetization is considered with allowance for circular precession, so that the closed system of equations that contains seven first-order equations and four boundary conditions is reduced to a squared system of four first-order equations in the absence of boundary conditions that corresponds to a model of two coupled oscillators with a cubic nonlinearity. It is demonstrated that the approximation provided by the squared system at a level of 20% remains to be correct up to a field strength of 0.4 of the saturation magnetization that corresponds to precession angles of the magnetization vector of up to 40°.

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INTRODUCTION

There has been significant interest in the excitation of ultrasonic oscillations using magnetostriction transducers [1–8]. Known applications (hydroacoustics, defectoscopy, and ultrasonic technologies) are supplemented with the applications of such transducers in the microwave acousto-electronics ($f = 10^9$–10^{11} Hz) in which relatively high mechanical Q factors of ferrite resonators (up to 10^7 for yttrium–iron garnet (YIG)) make it possible to construct high-efficiency devices for data processing [9, 10]. The most important problem involves the construction of a sufficiently efficient hypersonic emitter, which is impeded by the nonlinear parametric excitation of exchange spin waves that leads to significant loss even at an excitation level of 1 mW [11–13].

The results of [14–18] show that the parametric decay can be prevented using an appropriate configuration of the transducer. In the optimal configuration, a normally magnetized thin plate exhibits the lower frequency of ferromagnetic resonance (FMR) that coincides with the bottom of the spectrum of exchange spin waves. The absence of the parametric decay in such a configuration makes it possible to experimentally obtain precession angels of no less than 10°–20° for the magnetization vector [16–18] and provides additional possibilities in the excitation of high-power hypersound.

The theoretical analysis of the excitation of hypersound with the aid of a magnetoacoustic transducer based on a normally magnetized ferrite disk in the linear regime can be found in [9, 10, 19]. A nonlinear regime allows an increase in the excitation level of ultrasound by almost two orders of magnitude [20]. However, a cumbersome mathematical procedure leads to a significant increase in the computation time.

The above circumstances and practical demands necessitate the development of a simpler mathematical procedure that makes it possible to adequately solve the same problems with an accuracy that is sufficient for practical applications.

Such a procedure for the calculation of the excitation of hypersound using a magnetostriction transducer has been constructed in [21, 22] using a model of coupled oscillators in the linear [21] and quadratic [22] approximations. However, several examples have been considered under strict resonance conditions with disregard of the significantly nonlinear excitation that leads to self-modulation effects.

In this work, we develop the above model of the coupled oscillators based on the quadratic approximation to provide additional possibilities in the application in a wider frequency interval in the presence of a higher nonlinearity.

The work consists of two parts. In the first part, we derive and briefly analyze a system of coupled equations in the quadratic approximation with a linear approximation as a particular scenario. In the second part, the proposed procedure is used in the study of several nonlinear problems of the excitation of hypersound with the aid of a magnetostriction transducer.

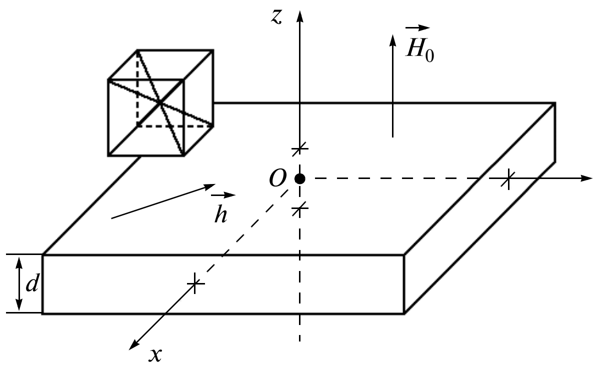

Fig. 1. Configuration of the system under study. The inset shows the scheme of crystallographic cell.

1. CONFIGURATION OF THE PROBLEM AND BASIC EQUATIONS

Figure 1 demonstrates the configuration of the problem that coincides with the configuration of [20–22]. The system is based on a plane-parallel plate with thickness d that exhibits magnetic, elastic, and magnetoelastic properties. The material of the plate has cubic crystallographic symmetry, and the (100) plane coincides with the plate plane.

External magnetic field $\vec{H}_0$ is exerted perpendicularly to the plate plane, and ac magnetic field $\vec{h}$ is oriented along the plate plane. The problem is solved in the Cartesian coordinates *Oxyz*, the *Oxy* plane of which coincides with the plate plane and the *Ox* , *Oy*, and *Oz* axes are parallel to the edges of the cubic crystallographic cell. Origin of coordinates *O* is located at the center of the plate, so that the plate planes have coordinates $z = \pm d/2$.

The basic system of the equations of motion for normalized components of magnetization $m_{x,y,z}$ is written as [20]

$$\frac{\partial m_x}{\partial t} = -\frac{\gamma}{1+\alpha^2} \times \left[\left(m_y + \alpha m_x m_z\right) H_z - \left(m_z - \alpha m_y m_x\right) H_y - \alpha\left(m_y^2 + m_z^2\right) H_x\right], \tag{1}$$

where γ is the gyromagnetic constant and α is the Hilbert decay parameter. The equations for $m_{y,z}$ are derived using cyclic change of variables x, y, and z.

Effective fields $H_{x,y,z}$ in these equations are given by

$$H_x = h_x + H_{ax}; \tag{2}$$

$$H_y = h_y + H_{ay}; \tag{3}$$

$$H_z = H_0 - 4\pi M_0 m_z + H_{az}, \tag{4}$$

where H_0 is the external static field, $h_{x,y}$ are the components of the external ac field, M_0 is the saturation magnetization of the plate material, and the expressions for field components $H_{ax,ay,az}$ are similar for the expressions from [20]

$$H_{ax} = -\frac{2K_0}{M_0} m_x - \frac{2K_1}{M_0} m_x \left(m_y^2 + m_z^2\right) - \frac{2K_2}{M_0} m_x m_y^2 m_z^2 - \frac{2B_1}{M_0} m_x \frac{\partial u_x}{\partial x} - \frac{B_2}{M_0} \times \left[m_y\left(\frac{\partial u_x}{\partial y} + \frac{\partial u_y}{\partial x}\right) + m_z\left(\frac{\partial u_z}{\partial x} + \frac{\partial u_x}{\partial z}\right)\right]. \tag{5}$$

Note that components H_{ay} and H_{az} are obtained using expression (5) with the aid of the cyclic change of variables x, y, and z. Here, $K_{0,1,2}$ are the constants of the uniaxial cubic anisotropy, $B_{1,2}$ are the constants of the magnetoelastic interaction, and $u_{x,y,z}$ are the components of the elastic displacement.

The equations for components of elastic displacement $u_{x,y}$ are represented as [20]

$$\frac{\partial^2 u_{x,y}}{\partial t^2} = -2\beta\frac{\partial u_{x,y}}{\partial t} + \frac{c_{44}}{\rho}\frac{\partial^2 u_{x,y}}{\partial z^2}; \tag{6}$$

and the boundary conditions are written as

$$c_{44}\left.\frac{\partial u_{x,y}}{\partial z}\right|_{z=\pm d/2} = -B_2 m_{z,y} m_z, \tag{7}$$

where β is the decay parameter, c_{44} is the elastic constant, and ρ is the density of the plate material.

Thus, we consider three first-order equations for the magnetization components and two second-order equations for the elastic-displacement components, which are equivalent to a system of seven first-order equations. The analysis of oscillations in such a system using the method of phase space [23–25] necessitates the determination of coordinates of singularities which is reduced to a solution of a seventh-order linear algebraic equation. The complexity of such a problem stimulates a search for simplifications. Below, we present several examples.

2. BASIC ASSUMPTIONS AND SHORTENED EFFECTIVE FIELDS

Using the approach of [21, 22], we use the simplifying assumptions in accordance with which the anisotropy is absent ($K_0 = 0$, $K_1 = 0$, and $H_2 = 0$), longitudinal elastic waves are absent ($B_1 = 0$), elastic displacements along the *Oy* axis are absent ($u_y = 0$), and elastic waves propagate only along the *Oz* axis ($\partial u_x/\partial x = 0$ and $\partial u_x/\partial y = 0$).

To simplify the notation, we introduce quantity

$$H_p = H_0 - 4\pi M_0. \tag{8}$$

Then, effective fields (2)–(4) are represented as

$$H_x = h_x - \frac{B_2}{M_0} m_z \frac{\partial u_x}{\partial z}; \quad (9)$$

$$H_y = h_y; \quad (10)$$

$$H_z = H_p + 4\pi M_0 - 4\pi M_0 m_z - \frac{B_2}{M_0} m_x \frac{\partial u_x}{\partial z}. \quad (11)$$

3. QUADRATIC APPROXIMATION

The condition for constancy of the length of magnetization vector leads to relationships [26, 27]

$$m_x^2 + m_y^2 + m_z^2 = 1. \quad (12)$$

We assume that $m_{x,y} \ll 1$ and expand quantity m_z in a Taylor series in the vicinity of unity up to quadratic terms with respect to quantities m_x and m_y:

$$m_z = 1 - \frac{1}{2}m_x^2 - \frac{1}{2}m_y^2. \quad (13)$$

4. EFFECTIVE FIELDS IN THE QUADRATIC APPROXIMATION

Substituting expression (13) in formulas (9)–(11) with allowance for the quadratic terms with respect to magnetization, we obtain the following effective fields:

$$H_x = h_x + \left(-\frac{B_2}{M_0} + \frac{B_2}{2M_0} m_x^2 + \frac{B_2}{2M_0} m_y^2\right)\frac{\partial u_x}{\partial z}; \quad (14)$$

$$H_y = h_y; \quad (15)$$

$$H_z = H_p + 2\pi M_0 m_x^2 + 2\pi M_0 m_y^2 - \frac{B_2}{M_0} m_x \frac{\partial u_x}{\partial z}. \quad (16)$$

These expressions contain coordinate derivative of the elastic displacement $\partial u_x/\partial z$. In accordance with formula (62) of [20], this quantity is represented as

$$\frac{\partial u_x}{\partial z} = -\frac{B_2}{c_{44}} m_x m_z + \frac{2}{d} v_x, \quad (17)$$

where v_x is the function that satisfies the following equation (formula (53) of [20]):

$$\frac{\partial^2 v_x}{\partial t^2} + 2\beta \frac{\partial v_x}{\partial t} + \frac{c_{44}\pi^2}{pd^2} v_x = \frac{4B_2 d}{c_{44}\pi^2}\left[\frac{\partial^2}{\partial t^2}(m_x m_z) + 2\beta \frac{\partial}{\partial t}(m_x m_z)\right]. \quad (18)$$

Function v_x is the solution to the boundary-value problem that is reduced to the first elastic mode [20]. Thus, we call it reduced function of elastic displacement or reduced elastic displacement. The x component of the elastic displacement is written as (formula (51) of [19])

$$u_x = -\frac{B_2}{c_{44}} m_x m_z z + v_x \sin\left(\frac{\pi}{d} z\right). \quad (19)$$

At the plate surfaces at $z = \pm d/2$, this quantity is

$$u_{xS} = \pm\left(-\frac{B_2 d}{2c_{44}} m_x m_z + v_x\right). \quad (20)$$

With allowance for quadratic approximation (13), derivative (17) is written as

$$\frac{\partial u_x}{\partial z} = -\frac{B_2}{c_{44}} m_x + \frac{B_2}{2c_{44}} m_x^3 + \frac{B_2}{2c_{44}} m_x m_y^2 + \frac{2}{d} v_x. \quad (21)$$

Using notation

$$b_0 = 2\pi M_0; \quad (22)$$

$$b_1 = \frac{B_2^2}{M_0 c_{44}}; \quad (23)$$

$$b_2 = \frac{B_2}{M_0 d} \quad (24)$$

and substituting expression (21) in formulas (14)–(16), we obtain the following effective fields:

$$H_x = h_x + b_1 m_x - b_1 m_x^3 - b_1 m_x m_y^2 - 2b_2 v_x + b_2 m_x^2 v_x + b_2 m_y^2 v_x; \quad (25)$$

$$H_y = h_y; \quad (26)$$

$$H_z = H_p + (b_0 + b_1) m_x^2 + b_0 m_y^2 - 2b_2 m_x v_x. \quad (27)$$

5. EQUATION FOR THE MAGNETIZATION IN THE QUADRATIC APPROXIMATION

We substitute formulas (25)–(27) in expression (1) with allowance for the fact that $b_1 \ll H_p$, $b_1 \ll b_2$, and $b_2 \sim H_p$ for real material parameters (e.g., parameters of YIG). Thus the multiplication of b_1 and $m_{x,y}^3$ yields quantities of the fourth order of smallness that are disregarded. On the assumption of a relatively low decay ($a \ll 1$), we obtain

$$\frac{\partial m_x}{\partial t} = -\gamma[-h_y + H_p m_y + b_0 m_x^2 m_y + b_0 m_y^3 - 2b_2 m_x m_y v_x + \alpha(H_p - b_1) m_x + 2\alpha b_2 v_x]; \quad (28)$$

$$\frac{\partial m_y}{\partial t} = \gamma[h_x + (H_p - b_1) m_x + b_0 m_x^3 + b_0 m_x m_y^2 + 2b_2 v_x - 4b_2 m_x^2 v_x - 2b_2 m_y^2 v_x - \alpha H_p m_y]. \quad (29)$$

We differentiate expression (28) with respect to time and substitute quantity $\partial m_y/\partial t$ using expression (29) to obtain

$$\begin{aligned}
&\frac{\partial^2 m_x}{\partial t^2} + \alpha\gamma(H_p - b_1)\frac{\partial m_x}{\partial t} + \gamma^2 H_p(H_p - b_1)m_x \\
&+ 2\gamma^2 H_p b_2 v_x + 2\alpha\gamma b_2\frac{\partial v_x}{\partial t} + 2\gamma^2 H_p b_0 m_x^3 \\
&- 2\gamma^2(3H_p - b_0)b_2 m_x^2 v_x - 4\gamma^2 b_2^2 m_x v_x^2 \\
&- \alpha\gamma^2 H_p^2 m_y + 2\gamma b_0 m_x m_y\frac{\partial m_x}{\partial t} - 2\gamma b_2 m_y v_x\frac{\partial m_x}{\partial t} \\
&- 2\gamma^2(H_p - 3b_0)b_2 m_y^2 v_x = \gamma^2 H_p h_x.
\end{aligned} \quad (30)$$

Several terms of this equation for magnetization component m_x contain component m_y. Thus, we must simultaneously analyze the equation for m_y. The equations for quantities m_x and m_y become independent in the approximation of circular precession.

6. APPROXIMATION OF CIRCULAR PRECESSION

We consider right-hand circular precession of the magnetization at frequency ω:

$$m_x = \cos(\omega t); \quad (31)$$

$$m_y = \sin(\omega t). \quad (32)$$

Thus, the following condition is satisfied:

$$m_y = -\frac{1}{\omega}\frac{dm_x}{dt}. \quad (33)$$

For the forced oscillations, precession frequency ω coincides with the external-force frequency. For free oscillations with relatively small amplitudes in the absence of magnetoelasticity, the frequency is given by [26, 27]

$$\omega = \gamma H_p. \quad (34)$$

7. EQUATION FOR MAGNETIZATION IN THE QUADRATIC APPROXIMATION FOR FORCED OSCILLATIONS

We consider forced oscillations of magnetization with frequency ω in the presence of external field

$$h_x = h_0\cos(\omega t). \quad (35)$$

Substituting expression (35) in expression (30) and collecting similar terms, we obtain

$$\begin{aligned}
&\frac{\partial^2 m_x}{\partial t^2} + \alpha\gamma\left(H_p - b_1 + \frac{\gamma H_p^2}{\omega}\right)\frac{\partial m_x}{\partial t} + \gamma^2 H_p(H_p - b_1)m_x \\
&+ 2\gamma^2 H_p b_2 v_x + 2\alpha\gamma b_2\frac{\partial v_x}{\partial t} + 2\gamma^2 H_p b_0 m_x^3 \\
&- 2\gamma^2(3H_p - b_0)b_2 m_x^2 v_x - 4\gamma^2 b_2^2 m_x v_x^2 \\
&+ \frac{2\gamma^2 b_0}{\omega^2}(2\gamma H_p - \omega)m_x\left(\frac{\partial m_x}{\partial t}\right)^2 \\
&+ \left(\frac{2\gamma b_2}{\omega}m_x\right)\left(\frac{\partial m_x}{\partial t}\right)\left(\frac{\partial v_x}{\partial t}\right) - \frac{2\gamma^2 b_0}{\omega^2}\Big[\gamma(H_p - 3b_0) - \omega\Big] \\
&\times\left(\frac{\partial m_x}{\partial t}\right)^2 v_x = \gamma^2 H_p h_0\cos(\omega t).
\end{aligned} \quad (36)$$

This equation is the second-order equation in terms of magnetization component m_x, and a similar independent equation can be obtained for component m_y using substitution of subscripts y for subscripts x.

8. EQUATION FOR ELASTIC DISPLACEMENT IN THE QUADRATIC APPROXIMATION

We consider Eq. (18) for reduced elastic displacement v_x. Substitution of derivatives (28) and (29) leads to cumbersome expressions. However, a test numerical calculation using Eqs. (1)–(7) that is similar to the calculation of [20] shows that the right-hand side of Eq. (18) is less than the left-hand side by more than an order of magnitude. Therefore, we can use $m_z = 1$ instead of expression (13). With allowance for the notation

$$c_1 = \frac{c_{44}\pi^2}{\rho d^2}; \quad (37)$$

$$c_2 = \frac{4B_2 d}{c_{44}\pi^2}, \quad (38)$$

Eq. (18) is represented as

$$\frac{\partial^2 v_x}{\partial t^2} + 2\beta\frac{\partial v_x}{\partial t} + c_1 v_x - 2\beta c_2\frac{\partial m_x}{\partial t} - c_2\frac{\partial^2 m_x}{\partial t^2} = 0. \quad (39)$$

Substituting the second derivative of magnetization (expression (36)) in Eq. (39), we derive

$$\begin{aligned}
&\frac{\partial^2 v_x}{\partial t^2} + 2(\beta + \alpha\gamma b_2 c_2)\frac{\partial v_x}{\partial t} + (c_1 + 2\gamma^2 H_p b_2 c_2)v_x \\
&+ \gamma^2 H_p(H_p - b_1)c_2 m_x + \left[\alpha\gamma\left(H_P - b_1 + \frac{\gamma H_p}{\omega}\right) - 2\beta\right] \\
&\times c_2\frac{\partial m_x}{\partial t} + 2\gamma^2 H_p b_0 c_2 m_x^3 - 2\gamma^2(3H_p - b_0)b_2 c_2 m_x^2 v_x \\
&- 4\gamma^2 b_2^2 c_2 m_x v_x^2 + \frac{2\gamma b_0 c_2}{\omega^2}(2\gamma H_p - \omega)m_x\left(\frac{\partial m_x}{\partial t}\right)^2 \\
&+ \frac{2\gamma b_2 c_2}{\omega}m_x\left(\frac{\partial m_x}{\partial t}\right)\left(\frac{\partial v_x}{\partial t}\right) - \frac{2\gamma b_2 c_2}{\omega^2}\Big[\gamma(H_p - 3b_0) - \omega\Big] \\
&\times\left(\frac{\partial m_x}{\partial t}\right)^2 v_x = \gamma^2 H_p c_2 h_0\cos(\omega t).
\end{aligned} \quad (40)$$

Equations (36) and (40) form the desired system of equations for magnetization and the reduced elastic displacement in the quadratic approximation. In this case, the total elastic displacements on the plate surfaces are given by formula (20).

9. EQUATIONS OF MOTION IN THE LINEAR APPROXIMATION

For comparison, we present the same system in the linear approximation. The equation for magnetization is written as

$$\frac{\partial^2 m_x}{\partial t^2} + \alpha\gamma(2H_P - b_1)\frac{\partial m_x}{\partial t} + \gamma^2 H_p(H_p - b_1)m_x + 2\gamma^2 H_p b_2 v_x + 2\alpha\gamma b_2 \frac{\partial v_x}{\partial \tau} = \gamma^2 H_p h_0 \cos(\omega t); \tag{41}$$

and the equation for the reduced elastic displacement is represented as

$$\frac{\partial^2 v_x}{\partial t^2} + 2(\beta + \alpha\gamma b_2 c_2)\frac{\partial v_x}{\partial t} + (c_1 + 2\gamma^2 H_p b_2 c_2) v_x + \gamma^2 H_p(H_p - b_1)c_2 m_x \times [\alpha\gamma(2H_p - b_1) - 2\beta]c_2 \frac{\partial m_x}{\partial t} = 0. \tag{42}$$

Up to notation, such a system coincides with the system of [21] (Eqs. 19 and 24).

For convenience, the system of equations (41) and (42) is called the linearized system and the system of equations (36) and (40) is called the squared system.

10. EQUATION FOR FREE OSCILLATIONS AT A RELATIVELY SMALL AMPLITUDE

For small-amplitude free oscillations (i.e., with disregard of the nonlinear mismatch), we can use expression (34) for frequency and substitute it in Eqs. (36) and (40). Thus, we derive the equation for magnetization

$$\begin{aligned}&\frac{\partial^2 m_x}{\partial t^2} + \alpha\gamma(2H_p - b_1)\frac{\partial m_x}{\partial t} + \gamma^2 H_p(H_p - b_1)m_x \\ &+ 2\gamma^2 H_p b_2 v_x + 2\alpha\gamma b_2 \frac{\partial v_x}{\partial t} + 2\gamma^2 H_p b_0 m_x^3 \\ &- 2\gamma^2(3H_p - b_0)b_2 m_x^2 v_x - 4\gamma^2 b_2^2 m_x v_x^2 \\ &+ \frac{2b_0}{H_p} m_x \left(\frac{\partial m_x}{\partial t}\right)^2 + \frac{2b_2}{H_p} m_x \left(\frac{\partial m_x}{\partial t}\right)\left(\frac{\partial v_x}{\partial t}\right) \\ &+ \frac{6b_0 b_2}{H_p^2} v_x \left(\frac{\partial m_x}{\partial t}\right)^2 = 0\end{aligned} \tag{43}$$

and the equation for the elastic displacement

$$\begin{aligned}&\frac{\partial^2 v_x}{\partial t^2} + 2(\beta + \alpha\gamma b_2 c_2)\frac{\partial v_x}{\partial t} + (c_1 + 2\gamma^2 H_p b_2 c_2) v_x \\ &+ \gamma^2 H_p(H_p - b_1)c_2 m_x + [\alpha\gamma(H_P - b_1) - 2\beta] \\ &\times c_2 \frac{\partial m_x}{\partial t} + 2\gamma^2 H_p b_0 c_2 m_x^3 - 2\gamma^2(3H_p - b_0)b_2 c_2 m_x^2 v_x \\ &- 4\gamma^2 b_2^2 c_2 m_x v_x^2 + \frac{2b_0 c_2}{H_p} m_x \left(\frac{\partial m_x}{\partial t}\right)^2 \\ &+ \frac{2b_2 c_2}{H_p} m_x \left(\frac{\partial m_x}{\partial t}\right)\left(\frac{\partial v_x}{\partial t}\right) + \frac{6b_0 b_2 c_2}{H_p^2}\left(\frac{\partial m_x}{\partial t}\right)^2 v_x = 0.\end{aligned} \tag{44}$$

These equations coincide with the equations of [22] up to the notation.

11. GENERAL SYSTEM OF EQUATIONS FOR FORCED OSCILLATIONS OF TWO COUPLED OSCILLATORS IN THE QUADRATIC APPROXIMATION

The squared system of equations (36) and (40) is a particular case of a more general symmetric system of two second-order nonlinear differential equations in terms of variables x_1 and x_2 that correspond to nonlinear forced oscillations of a system of two coupled oscillators with two degrees of freedom:

$$\begin{aligned}&p_{11}\frac{\partial^2 x_1}{\partial t^2} + p_{12}\frac{\partial x_1}{\partial t} + p_{13}x_1 + q_{11}x_2 + q_{12}\frac{\partial x_2}{\partial t} \\ &+ r_{11}x_1^3 + r_{12}x_1^2 x_2 + r_{13}x_1 x_2^2 + r_{14}x_2^3 \\ &+ s_{11}x_1\left(\frac{\partial x_1}{\partial t}\right)^2 + s_{12}x_1\left(\frac{\partial x_1}{\partial t}\right)\left(\frac{\partial x_2}{\partial t}\right) + s_{13}x_1\left(\frac{\partial x_2}{\partial t}\right)^2 \\ &+ s_{14}x_2\left(\frac{\partial x_1}{\partial t}\right)^2 + s_{15}x_2\left(\frac{\partial x_1}{\partial t}\right)\left(\frac{\partial x_2}{\partial t}\right) + s_{16}x_2\left(\frac{\partial x_2}{\partial t}\right)^2 \\ &= A_{11}\cos(\omega t) + A_{12}\sin(\omega t);\end{aligned} \tag{45}$$

$$\begin{aligned}&p_{21}\frac{\partial^2 x_2}{\partial t^2} + p_{22}\frac{\partial x_2}{\partial t} + p_{23}x_2 + q_{21}x_1 + q_{22}\frac{\partial x_1}{\partial t} \\ &+ r_{21}x_2^3 + r_{22}x_2^2 x_1 + r_{23}x_2 x_1^2 + r_{24}x_1^3 \\ &+ s_{21}x_2\left(\frac{\partial x_2}{\partial t}\right)^2 + s_{22}x_2\left(\frac{\partial x_2}{\partial t}\right)\left(\frac{\partial x_1}{\partial t}\right) + s_{23}x_2\left(\frac{\partial x_1}{\partial t}\right)^2 \\ &+ s_{24}x_1\left(\frac{\partial x_2}{\partial t}\right)^2 + s_{25}x_1\left(\frac{\partial x_2}{\partial t}\right)\left(\frac{\partial x_1}{\partial t}\right) + s_{26}x_1\left(\frac{\partial x_1}{\partial t}\right)^2 \\ &= A_{21}\cos(\omega t) + A_{22}\sin(\omega t).\end{aligned} \tag{46}$$

These equations are transformed into each other using interchange of subscripts 1 and 2. A more general representation of such a coupled system must contain the terms that consist of products of three derivatives. However, such terms are missing in the above problem of the forced magnetoelastic oscillations in a ferrite plate and are not taken into account in the analysis.

The linearized system of Eqs. (41) and (42) is a particular case of such a system.

We represent coefficients of the system of equations (45) and (46) in terms of material parameters, geometrical parameters of the magnetic plate, and external field:

$$p_{11} = 1; \quad (47)$$

$$p_{12} = \alpha\gamma\left(H_p - b_1 + \frac{\gamma H_p^2}{\omega}\right); \quad (48)$$

$$p_{13} = \gamma^2 H_p (H_p - b_1); \quad (49)$$

$$q_{11} = 2\gamma^2 H_p b_2; \quad (50)$$

$$q_{12} = 2\alpha\gamma b_2; \quad (51)$$

$$r_{11} = 2\gamma^2 H_p b_0; \quad (52)$$

$$r_{12} = -2\gamma^2 (3H_p - b_0) b_2; \quad (53)$$

$$r_{13} = -4\gamma^2 b_2^2; \quad (54)$$

$$r_{14} = 0; \quad (55)$$

$$s_{11} = \frac{2\gamma b_0}{\omega^2}(2\gamma H_p - \omega); \quad (56)$$

$$s_{12} = \frac{2\gamma b_2}{\omega}; \quad (57)$$

$$s_{13} = 0; \quad (58)$$

$$s_{14} = -\frac{2\gamma b_2}{\omega^2}[\gamma(H_p - 3b_0) - \omega]; \quad (59)$$

$$s_{15} = 0; \quad (60)$$

$$s_{16} = 0; \quad (61)$$

$$A_{11} = \gamma^2 H_p h_0; \quad (62)$$

$$A_{12} = 0;; \quad (63)$$

$$p_{21} = 1; \quad (64)$$

$$p_{22} = 2(\beta + \alpha\gamma b_2 c_2); \quad (65)$$

$$p_{23} = c_1 + 2\gamma^2 H_p b_2 c_2; \quad (66)$$

$$q_{21} = \gamma^2 H_p (H_p - b_1) c_2; \quad (67)$$

$$q_{22} = \left[\alpha\gamma\left(H_p - b_1 + \frac{\gamma H_p}{\omega}\right) - 2\beta\right] c_2; \quad (68)$$

$$r_{21} = 0; \quad (69)$$

$$r_{22} = -4\gamma^2 b_2^2 c_2; \quad (70)$$

$$r_{23} = -2\gamma^2 (3H_p - b_0) b_2 c_2; \quad (71)$$

$$r_{24} = 2\gamma^2 H_p b_0 c_2; \quad (72)$$

$$s_{21} = 0; \quad (73)$$

$$s_{22} = 0; \quad (74)$$

$$s_{23} = -\frac{2\gamma b_2 c_2}{\omega^2}[\gamma(H_p - 3b_0) - \omega]; \quad (75)$$

$$s_{24} = 0; \quad (76)$$

$$s_{25} = \frac{2\gamma b_2 c_2}{\omega}; \quad (77)$$

$$s_{26} = \frac{2\gamma b_0 c_2}{\omega^2}(2\gamma H_P - \omega); \quad (78)$$

$$A_{21} = \gamma^2 H_P c_2 h_0; \quad (79)$$

$$A_{22} = 0. \quad (80)$$

In these expressions, we use the following notation

$$H_p = H_0 - 4\pi M_0; \quad (81)$$

$$b_0 = 2\pi M_0; \quad (82)$$

$$b_1 = \frac{B_2^2}{M_0 c_{44}}; \quad (83)$$

$$b_2 = \frac{B_2}{M_0 d}; \quad (84)$$

$$c_1 = \frac{c_{44}\pi^2}{\rho d^2}; \quad (85)$$

$$c_2 = \frac{4B_2 d}{c_{44}\pi^2}. \quad (86)$$

12. ANALYSIS OF THE STRUCTURE OF THE SYSTEM FOR COUPLED OSCILLATORS

We consider the structure of the system of Eqs. (45) and (46) that contains a relatively large number of terms. To estimate the contribution of these terms to formation of oscillations of the magnetization and elastic displacement, we estimate relative values of terms for a typical scenario of the excitation of ultrasonic oscillations. We use the material parameters of YIG from [9, 26, 27]: $4\pi M_0 = 1750$ G; $B_2 = 6.96 \times 10^6$ erg cm^{-3}; $c_{44} = 7.64 \times 10^{11}$ erg cm^{-3}; $\rho = 5.17$ g cm^{-3}; decay parameters, $\alpha = 0.02$ and $\beta = 10^9$ s^{-1}, film thickness, $d = 6.865 \times 10^{-5}$ cm; field, $H_0 = 2750$ Oe; and frequency, $f = 2800$ MHz ($\omega = 1.7592 \times 10^{10}$ s^{-1}). The frequency and field correspond to the excitation of the ferromagnetic and elastic resonances. Additional parameters given by expressions (81)–(86) are as fol-

Table 1. Terms of the system of equations (45) and (46) at $B_2 = 6.96 \times 10^6$ erg cm^{-3}

First oscillator			Second oscillator		
terms	$h_0 = 0.01$	$h_0 = 400$	terms	$h_0 = 0.01$	$h_0 = 400$
$p_{11}\omega^2 x_1$	6.19×10^{16}	1.86×10^{20}	$p_{21}\omega^2 x_2$	1.55×10^8	3.09×10^{11}
$p_{12}\omega x_1$	2.49×10^{15}	7.46×10^{18}	$p_{22}\omega x_2$	1.76×10^7	3.52×10^{10}
$p_{13}x_1$	6.23×10^{16}	1.87×10^{20}	$p_{23}x_2$	1.55×10^8	3.09×10^{11}
$q_{11}x_2$	2.27×10^{14}	4.54×10^{17}	$q_{21}x_1$	1.58×10^7	4.73×10^{10}
$q_{12}\omega x_2$	4.52×10^{12}	9.04×10^{15}	$q_{22}\omega x_1$	1.47×10^6	4.40×10^9
$r_{11}x_1^3$	4.36×10^9	1.18×10^{20}	$r_{21}x_2^3$	0.00	0.00
$r_{12}x_1^2 x_2$	1.93×10^7	3.47×10^{17}	$r_{22}x_2^2 x_1$	8.37×10^{-6}	1.00×10^5
$r_{13}x_1 x_2^2$	3.30×10^4	3.96×10^{14}	$r_{23}x_2 x_1^2$	4.90×10^{-3}	8.80×10^7
$r_{14}x_2^3$	0.00	0.00	$r_{24}x_1^3$	1.10×10^0	2.98×10^{10}
$s_{11}\omega^2 x_1^3$	4.38×10^9	1.18×10^{20}	$s_{21}\omega^2 x_2^3$	0.00	0.00
$s_{12}\omega^2 x_1^2 x_2$	9.04×10^6	1.63×10^{17}	$s_{22}\omega^2 x_2^2 x_1$	0.00	0.00
$s_{13}\omega^2 x_1 x_2^2$	0.00	0.00	$s_{23}\omega^2 x_2 x_1^2$	6.00×10^{-3}	1.08×10^8
$s_{14}\omega^2 x_1^2 x_2$	2.28×10^7	4.28×10^{17}	$s_{24}\omega^2 x_2^2 x_1$	0.00	0.00
$s_{15}\omega^2 x_1 x_2^2$	0.00	0.00	$s_{25}\omega^2 x_2 x_1^2$	2.30×10^{-3}	4.12×10^7
$s_{16}\omega^2 x_2^3$	0.00	0.00	$s_{26}\omega^2 x_1^3$	1.11×10^0	3.00×10^{10}
A_{11}	3.11×10^{15}	1.25×10^{20}	A_{21}	7.90×10^5	3.16×10^{10}
A_{12}	0.00	0.00	A_{22}	0.00	0.00

lows: $H_p = 1000$ Oe, $b_0 = 875$ G, $b_1 = 0.4553$ Oe, $b_2 = 7.2802 \times 10^8$ Oe cm^{-1}, $c_1 = 3.0947 \times 10^{20}$ s^{-1}, and $c_2 = 2.5346 \times 10^{-10}$ cm.

The terms in Eqs. (45) and (46) represent products of coefficients a_{ik}, b_{ik} and c_{ik}, variables m and v raised to different powers, and frequency ω and squared frequency (that result from time differentiation of function $\exp(i\omega t)$).

Table 1 presents the absolute values of the terms of Eqs. (45) and (46) for two external fields (the signs and dimensions in the CGS system are omitted for simplicity).

Using the solution to the closed problem (Eqs. (1)–(7)), we obtain the following amplitudes of variables: $m = 2 \times 10^{-4}$ (6×10^{-1}) and $v = 5 \times 10^{-13}$ (1×10^{-9}) cm for the field $h_0 = 0.01$ (400) Oe.

We choose such fields due to the following reasons.

Field $h_0 = 0.01$ Oe corresponds to the linear regime, so that we obtain a reference point that can be used to study the role of nonlinearity.

Field $h_0 = 400$ Oe corresponds to a strongly nonlinear regime in which the quadratic approximation yields an error of about 5% with respect to the magnetization and a factor of 2 with respect to the elastic displacement [22].

Table 1 shows that the contribution of the terms with coefficients r_{ik} and s_{ik} in the linear regime ($h_0 = 400$ Oe) is less than the contribution of the terms with coefficients p_{ik} and q_{ik} by more than seven orders of magnitude. Thus, the former contributions can be neglected. The coupling is provided by the terms that are proportional to the first power of variables (i.e., q_{11} and q_{21}), whereas the terms that are proportional to the derivatives of variables with coefficients q_{12} and q_{22} provide the contribution that is less than the contribution of decay parameters p_{12} and p_{22} by one-to-three orders of magnitude.

In the strongly nonlinear regime ($h_0 = 400$ Oe), the contribution of the nonlinear terms with coefficients r_{11} and r_{24} significantly increases. Such an increase corresponds to the role of nonlinearity of the magnetic oscillator related to the magnetization nonlinearity. The contribution of the coupling provided by the linear terms with coefficients q_{ik} slightly increases and becomes greater by an order of magnitude. Note significant contributions of the nonlinear terms with derivatives, in particular, with coefficient s_{11} for the magnetic oscillator and coefficient s_{26} for the elastic oscillator. In both cases, the nonlinearity is provided by the nonlinearity of the magnetic oscillator. The nonlinear coupling via derivatives is noticeably enhanced owing to the terms with coefficients s_{12} and s_{14} for the magnetic oscillator and s_{23} and s_{25} for the elastic oscillator. In both cases, the nonlinearity is provided by the squared amplitude of the magnetic oscillator.

Thus, the contribution of the terms with derivatives increases in the strongly nonlinear regime owing to the magnetic oscillator. The terms that are proportional to the amplitude of the magnetization oscillations in the second and third powers must be taken into account in the equations.

Constant of magnetoelastic interaction B_2 must be relatively large for the efficient excitation of the hypersonic oscillations. For example, the constant of the terbium–iron garnet is greater than the YIG constant by a factor of about 5 [9]. The results of [28] show additional nonlinear effects, for example, significant variations in the field of the orientational transition of magnetization and spontaneous reorientation of the magnetization vector at constant B_2 that is higher than the critical level.

In this regard, it is expedient to analyze the terms of Eqs. (45) and (46) at relatively large values of the constant. Table 2 presents the results for $B_2 = 20 \times B_2$ (YIG). The results are obtained for the parameters of Table 1 except for constant B_2. Additional parameters (81)–(86) are as follows: $H_p = 1000$ Oe, $b_0 = 875$ G, $b_1 = 182.12$ Oe, $b_2 = 1.4560 \times 10^{10}$ Oe cm^{-1}, $c_1 = 3.0947 \times 10^{20}$ s^{-1}, and $c_2 = 5.0692 \times 10^{-9}$ cm.

The comparison of the results of Tables 1 and 2 shows an increase in the contribution of the linear terms determined by coefficients q_{ik} at a relatively large constant B_2 even in the linear regime ($h_0 = 0.01$ Oe). The linear terms become comparable with the terms determined by coefficients p_{ik} (the terms with coefficient q_{11} are even greater than the main terms).

The contribution of the terms with derivatives significantly increases in the nonlinear regime. For the magnetic oscillator such an increase takes place for all terms with coefficients r_{ik} and s_{ik}. Such terms are less than the main terms with coefficients p_{ik} and q_{ik} by only an order of magnitude, and the term that is responsible for the decay with coefficient p_{12} is even greater. For the elastic oscillator, the main contribution is provided by the terms with coefficients r_{23} and s_{23} that are proportional to the squared amplitude of the magnetic oscillations.

Thus, the contribution of the coupling constants is significant at a relatively large constant of the magnetoelastic interaction even in the linear regime and the contribution additionally increases in the nonlinear regime. In the strongly nonlinear regime of the magnetic oscillator, significant contributions are provided by all terms with derivatives. For the elastic terms, relatively large contributions are provided by terms that are proportional to the squared amplitude of the magnetization oscillations.

13. SHORTENED EQUATIONS THAT CONTAIN THE MAIN TERMS

Equations (45) and (46) can be simplified using the results of Tables 1 and 2 that make it possible to choose the most significant terms. We separately represent the shortened equations for small and large constants B_2.

For small constant B_2, we obtain the equations for the first oscillator

$$p_{11}\frac{\partial^2 x_1}{\partial t^2} + p_{12}\frac{\partial x_1}{\partial t} + p_{13}x_1 + q_{11}x_2 + r_{11}x_1^3 + s_{11}x_1\left(\frac{\partial x_1}{\partial t}\right)^2 = A_{11}\cos(\omega t) \tag{87}$$

and the second oscillator

$$p_{21}\frac{\partial^2 x_2}{\partial t^2} + p_{22}\frac{\partial x_2}{\partial t} + p_{23}x_2 + q_{21}x_1 + q_{22}\frac{\partial x_1}{\partial t} + r_{24}x_1^3 + s_{26}x_1\left(\frac{\partial x_1}{\partial t}\right)^2 = A_{21}\cos(\omega t). \tag{88}$$

For large constant B_2, we derive the equations for the first oscillator

$$p_{11}\frac{\partial^2 x_1}{\partial t^2} + p_{12}\frac{\partial x_1}{\partial t} + p_{13}x_1 + q_{11}x_2 + q_{12}\frac{\partial x_2}{\partial t} + r_{11}x_1^3 + r_{12}x_1^2x_2 + r_{13}x_1x_2^2 + s_{11}x_1\left(\frac{\partial x_1}{\partial t}\right)^2 + s_{12}x_1\left(\frac{\partial x_1}{\partial t}\right)\left(\frac{\partial x_2}{\partial t}\right) + s_{14}x_2\left(\frac{\partial x_1}{\partial t}\right)^2 = A_{11}\cos(\omega t) \tag{89}$$

and the second oscillator

$$p_{21}\frac{\partial^2 x_2}{\partial t^2} + p_{22}\frac{\partial x_2}{\partial t} + p_{23}x_2 + q_{21}x_1 + q_{22}\frac{\partial x_1}{\partial t} + r_{23}x_2x_1^2 + s_{23}x_2\left(\frac{\partial x_1}{\partial t}\right)^2 = A_{21}\cos(\omega t). \tag{90}$$

Table 2. Terms of the system of equations (45) and (46) at $B_2 = 20\ B_2$ (YIG)

First oscillator			Second oscillator		
terms	$h_0 = 0.01$	$h_0 = 400$	terms	$h_0 = 0.01$	$h_0 = 400$
$p_{11}\omega^2 x_1$	9.28×10^{14}	4.64×10^{19}	$p_{21}\omega^2 x_2$	4.02×10^{7}	1.86×10^{12}
$p_{12}\omega x_1$	3.39×10^{13}	1.70×10^{18}	$p_{22}\omega x_2$	4.69×10^{6}	2.17×10^{11}
$p_{13}x_1$	7.65×10^{14}	3.82×10^{19}	$p_{23}x_2$	4.62×10^{7}	2.13×10^{12}
$q_{11}x_2$	1.18×10^{15}	5.44×10^{19}	$q_{21}x_1$	3.88×10^{6}	1.94×10^{11}
$q_{12}\omega x_2$	2.35×10^{13}	1.08×10^{18}	$q_{22}\omega x_1$	4.58×10^{5}	2.29×10^{10}
$r_{11}x_1^3$	1.47×10^{4}	1.84×10^{18}	$r_{21}x_2^3$	0.00	0.00
$r_{12}x_1^2x_2$	2.26×10^{4}	2.60×10^{18}	$r_{22}x_2^2x_1$	6.79×10^{-5}	7.23×10^{9}
$r_{13}x_1x_2^2$	1.34×10^{4}	1.43×10^{18}	$r_{23}x_2x_1^2$	1.14×10^{-4}	1.32×10^{10}
$r_{14}x_2^3$	0.00	0.00	$r_{24}x_1^3$	7.46×10^{-5}	9.33×10^{9}
$s_{11}\omega^2x_1^3$	1.48×10^{4}	1.85×10^{18}	$s_{21}\omega^2x_2^3$	0.00	0.00
$s_{12}\omega^2x_1^2x_2$	1.06×10^{4}	1.22×10^{18}	$s_{22}\omega^2x_2^2x_1$	0.00	0.00
$s_{13}\omega^2x_1x_2^2$	0.00	0.00	$s_{23}\omega^2x_2x_1^2$	1.41×10^{-4}	1.63×10^{10}
$s_{14}\omega^2x_1^2x_2$	2.78×10^{4}	3.21×10^{18}	$s_{24}\omega^2x_2^2x_1$	0.00	0.00
$s_{15}\omega^2x_1x_2^2$	0.00	0.00	$s_{25}\omega^2x_2x_1^2$	5.36×10^{-5}	1.19×10^{9}
$s_{16}\omega^2x_2^3$	0.00	0.00	$s_{26}\omega^2x_1^3$	7.49×10^{-5}	9.36×10^{9}
A_{11}	3.11×10^{15}	1.25×10^{20}	A_{21}	1.58×10^{7}	6.32×10^{11}
A_{12}	0.00	0.00	A_{22}	0.00	0.00

In both cases, the system of equations is not totally symmetric due to different natures of the magnetic and elastic oscillation systems under study. Note that the remaining linear terms contain variables and derivatives of variables and the remaining nonlinear terms contain the variables in the third power and the products of variables and squared derivatives (i.e., cubic terms).

14. VERIFICATION OF THE CORRECTNESS OF THE QUADRATIC APPROXIMATION

We consider the closeness of the results provided by the squared system of equations (45) and (46) to the results of the original nonlinear system (1)–(7). We also consider the approximation that is provided by the linearized system of equations (41) and (42).

Figure 2 shows the dependences of the transverse component of magnetization (a) and elastic displacement (b) on the ac-field amplitude for the YIG parameters from [21]. Displacement u is obtained from variable v using formula (20) at $z = d/2$.

It is seen that both (squared and linearized) systems slightly overestimate the magnetization and displacement amplitudes and the overestimation is greater for the displacement. The approximation provided by the squared system is significantly better than the approximation of the linearized system.

In particular, the linearized system of equations (41) and (42) describes the dependences up to ac-field amplitude of 5 Oe ($h_0/M_0 = 0.04$) with an accuracy of about 10%. When the amplitude is less than 10 Oe ($h_0/M_0 = 0.07$), the accuracy is about 30%. Then, the approximation becomes inaccurate.

The squared system of equations (45) and (46) describes the amplitude of magnetic oscillations with an accuracy of about 3% when the ac-field amplitude is less

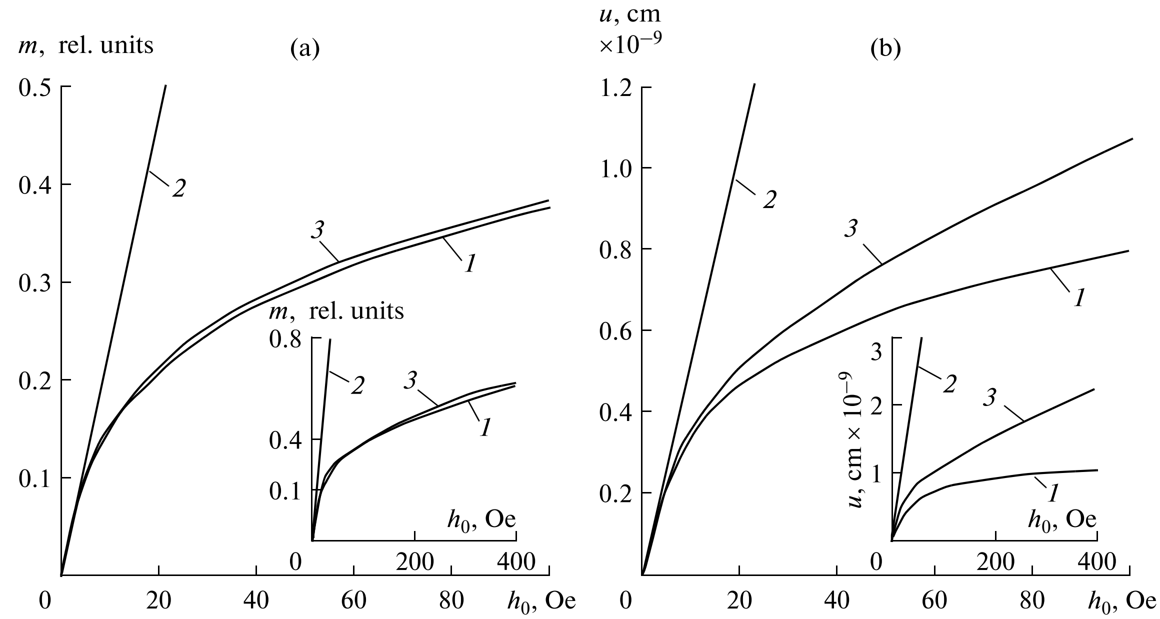

Fig. 2. Plots of (a) transverse component of magnetization and (b) elastic displacement vs. ac-field amplitude: (*1*) solution to closed system of equations (1)–(7), (*2*) solution to linearized system of equations (41) and (42), (*3*) solution to squared system of equations (45) and (46) for the material parameters of YIG ($4\pi M_0 = 1750$ G, $B_2 = 6.96 \times 10^6$ erg cm^{-3}, $c_{44} = 7.64 \times 10^{11}$ erg cm^{-3}, $\rho = 5.17$ g cm^{-3}, $\alpha = 0.02$, and $\beta = 10^9$ s^{-1}), $d = 6.865 \times 10^{-5}$ cm, $H_0 = 2750$ Oe, and an excitation frequency of 2.8 GHz. The insets show the same curves at ac field of up to 400 Oe.

than 100 Oe ($h_0/M_0 = 0.71$). The accuracy is about 5% if the field amplitude is less than 400 Oe $(h_0/M_0 = 2.86)$. The same system describes the amplitude of elastic oscillations with an accuracy of 20% when the ac-field amplitude is less than 60 Oe $(h_0/M_0 = 0.43)$. For amplitudes of less than 100 and 400 Oe, the accuracies are 25 and 100%, respectively.

For an accuracy of 20% in the analysis of elastic oscillations, the squared system makes it possible to consider amplitudes of ac field of up to 0.40 of the saturation magnetization, so that that the corresponding precession angles are no greater than 25°. For the linearized system, the same parameters are 0.05 and 3°, respectively. In the study of the magnetic oscillations, the squared system can be used when the fields are greater than the saturation magnetization by a factor of no less than 3 and the precession angle is up to 30°–40°. We assume that such precession angles allow the application of the squared system in the analysis of nonlinear processes with a relatively high accuracy.

15. CORRECTNESS OF THE APPROXIMATION AT A RELATIVELY LARGE CONSTANT OF MAGNETOELASTIC INTERACTION

Constant of magnetoelastic interaction B_2 that is slightly greater than the YIG constant is important under certain conditions for the excitation of elastic oscillations (e.g., for reorientation of the magnetization vector [28]). Therefore, it is of interest to consider the accuracies provided by the linear and quadratic approximations.

Figure 3 presents the dependences of the oscillation amplitudes of magnetization (a) and elastic displacement (b) on constant ratio $N = B_2/B_2$ (YIG) for different excitation levels (including the level that corresponds to the strongly nonlinear regime).

Figure 3a shows that the accuracies of both approximation are about 20% for the magnetization oscillations at a relatively low excitation level (group of curves *1* corresponding to $h_0 = 10$ Oe) and $N \sim 1$. The accuracy is about 1% when ratio N increases to 10, and an additional increase in ratio N leads to better accuracies.

At the intermediate excitation level (group of curves *2* corresponding to $h_0 = 100$ Oe), an order-of-magnitude deviation is obtained for the linear approximation at $N \sim 1$ whereas the error for the quadratic approximation is no greater than 10%. The errors for both approximations decrease with an increase in ratio N, so that an increase in ratio from 1 to 20 leads to a decrease in the error from 10 to 1%.

At the high excitation level (group of curves *3* corresponding to $h_0 = 400$ Oe), the error of the linear approximation at relatively small N is greater than one and a half order of magnitude whereas the error for the quadratic approximation is no greater than 20%. When ratio N increases to 20, the errors of both

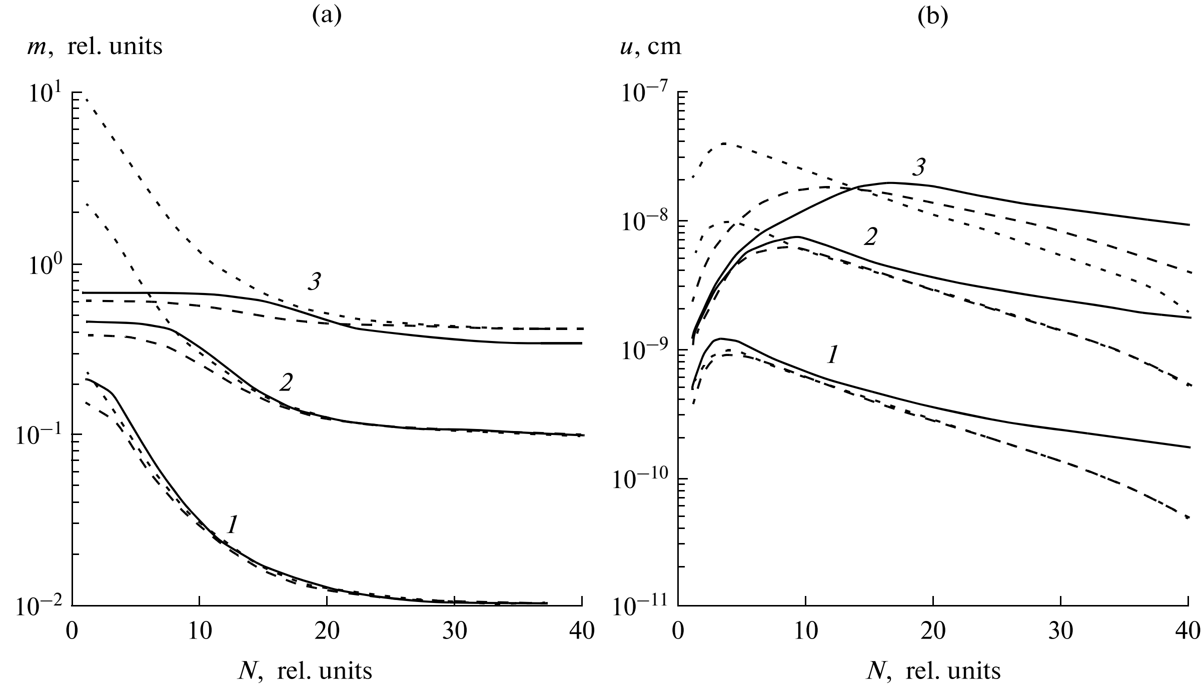

Fig. 3. Plots of oscillation amplitudes of (a) magnetization and (b) elastic displacement vs. ratio of constants of magnetoelastic interaction $N = B_2/B_2$ (YIG) for excitation fields $h_0 = (1)$ 10, (2) 100, and (3) 400 Oe: (solid lines) solution to closed system of equations (1)–(7), (dotted line) linear approximation using equations (41) and (42), and (dashed line) quadratic approximation using equations (45) and (46).

approximations decrease to about 20%. The errors remain unchanged if ratio N additionally increases.

For the elastic displacement (Fig. 3b), the greatest deviations of the approximate solution are observed at relatively small ratios $N \sim 2$–3. The deviations increase with an increase in the excitation level, so that the errors are about 20% for both approximations at $h_0 = 10$ Oe (curves *1*). For $h_0 = 400$ Oe (curves *3*), the error for the linear approximation is about an order of magnitude and the error for the quadratic approximation is no greater than 30%.

When ratio N increases to 20, the errors of both approximations decrease to 10% at $h_0 = 10$ Oe. The errors are no greater than 30% at $h_0 = 400$ Oe. A further increase in ratio N leads to an increase in the errors: the errors are 30% at $N = 40$ and $h_0 = 10$ Oe, and the errors for the linear and quadratic approximations are 80 and 30%, respectively, at $h_0 = 400$ Oe.

We conclude that the accuracies of both linear and quadratic approximations for the oscillations of magnetization increase with an increase in constant of magnetoelastic interaction B_2 even at relatively high nonlinearities. When constant B_2 increases, the accuracy for elastic oscillations, first, increases, reaches maximum, and, then, decreases. The maximum accuracy corresponds to constant B_2 that is greater than the constant for YIG by an order of magnitude.

CONCLUSIONS

The following results are obtained in the first part of the work.

The excitation of hypersonic oscillations by ac magnetic field in a plane-parallel normally magnetized ferrite plate is considered in the framework of the problem of the magnetostriction microwave transducer working at the frequency of the ferromagnetic resonance. The equations of motion of the magnetization and elastic-displacement vectors are derived with allowance for initial conditions and excitation by ac magnetic field.

To simplify the problem, we consider the quadratic approximation with respect to magnetization with allowance for the circular precession. Thus, the closed system of equations that contains seven first-order equations and four boundary conditions is reduced to a squared system of four first-order equations without boundary conditions. The linearized system that has been derived in previous works is a particular case of the squared system.

We introduce equivalent parameters of the squared system that are represented in terms of material parameters and geometrical parameters of the original ferrite plate. A generalized symmetric representation is obtained for the system of equations in the quadratic approximation. The system of equations corresponds to a model system of two coupled oscillators in which the nonlinearity is described using the third-order terms with respect to magnetization, elastic displacement, and their derivatives.

Shortened equations that contain only significant terms responsible for the dynamics of the system are derived using the numerical analysis of the relative contributions of equivalent parameters for YIG. It is demonstrated that the main contribution is provided by the terms proportional to the magnetization in the third power and the product of magnetization and its squared time derivative.

The analysis of the time evolution of oscillations makes it possible to compare the approximations provided by the linearized and squared systems and the solution to the original nonlinear system. In the study of elastic oscillations with an accuracy of 20%, the linearized system is correct at ac fields of no greater than 0.05 of the saturation magnetization at precession angles of about 3°. The squared system remains correct up to fields of 0.40 of the saturation magnetization corresponding to precession angles of up to 25°. With respect to magnetic oscillations, the squared system exhibits an accuracy of 5% for the fields that are greater than the saturation magnetization by a factor of more than 3 (the corresponding precession angles are up to 40°).

We consider the accuracy of calculations using the linearized and squared systems at relatively high constants of magnetoelastic interaction. For the oscillations of magnetization, an increase in the ratio of the constant of magnetoelastic interaction to the YIG constant to 40 leads to the accuracies of the linear and quadratic approximations that are no worse than 5% when the field is no greater than 0.40 of the saturation magnetization. For the elastic oscillations, the accuracy is 40% for the constant than is greater than the YIG constant by an order of magnitude. Then, the maximum accuracy (10%) is reached, and a further increase in the constant to a constant ratio of 40 leads to a decrease in the accuracy to a level of 80%.

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